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Islamic Historical Review on the Middle Age Lunar Crescent Visibility Criterion

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Abstract

Middle Age lunar crescent visibility criterion is criterion that was produced during 8th until 17th century. This includes al-Khawarizmi lunar crescent visibility criterion in 8th century until al-Lathiqi lunar crescent visibility criterion in 17th century. Numbers of review on mathematics and astronomy during the Middle Age, however the number of review that specifically written for Middle Age lunar crescent visibility criterion limited, with majority of review is written to study the historical of Middle Age science, astronomy, mathematics and geography as a whole, and not converge on lunar crescent visibility criterion. . Therefore, this article aimed to provide a review on Middle Age lunar crescent visibility criterion. The review is conducted using literature analysis, snowball literature search and specific inclusion and exclusion criteria. The review is performed based on 13 works on lunar crescent visibility criterion that pass the selection criteria. The review found out that most of the lunar crescent visibility criterion is based on al-Khawarizmi arc of separation and solar longitude, lunar crescent visibility criterion, with exception on Ibn Tariq, Ibn Qurra and Ibnu Yunus lunar crescent visibility criterion that adopt angular distance or elongation. The review suggest that a new outlook on Middle Age lunar crescent visibility criterion study can be done, by conducting an assessment on new data of moonsighting, and comparing with modern lunar crescent visibility criterion research.

Keywords: Middle Age, Lunar Crescent, Visibility, Criterion, Islamic Calendar.

Introduction

Lunar crescent visibility criterion is a criterion to predict a visibility of lunar crescent after it pass conjunction during its lunar crescent. The criterion is based on the records of moonsighting and design around the parameter and value that denotes a sighting of a lunar crescent. Lunar crescent visibility is either based on theoretical or empirical approaches. An empirical lunar crescent visibility criterion is a criterion that is built directly from data of moonsighting.[1] Its parameter such as elongation or altitude can be traced straight to any records of moonsighting. Such example of an empirical lunar crescent visibility criterion is Danjon,[2] Odeh[3] and recently MABIMS 2021.[4] A theoretical lunar crescent visibility criterion is a criterion that is built upon theoretical discussion on visibility of a lunar crescent and cannot directly traced to any moonsighting records.[5] Theoretical discussion of a lunar crescent visibility criterion could include contrast threshold such as Sultan and Schaefer,[6] extrapolation of moonsighting data such as Amir[7] or complex mathematical approach such as Ahmad.[8]

Research on lunar crescent visibility criterion is prevalent among Muslim because the new Islamic Month require sighting of a new moon after sunset of 29th Hijri Month. Should a new moon sighted after sunset of 29th Hijri month, next day would be commenced as a new Hijri month. If the new moon

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[1] Mohammad Ilyas (1997), *Sistem Kalendar Islam dari Perspektif Astronomi*, Kuala Lumpur: Dewan Bahasa dan Pustaka, p. 31.
[2] A. Danjon (1936), "Le Croissant Lunaire," *L'Astronomie: Bulletin de La Société Astronomique de France*, Vol. 50, pp. 57–65.
[3] Mohammad Sh. Odeh (2004), "New Criterion for Lunar Crescent Visibility," *Experimental Astronomy*, Vol. 18, No. 1, pp. 39–64.
[4] JAKIM (2019), "Pertemuan Pakar Falak MABIMS 2019," accessed 18 April 2022, https://www.islam.gov.my/en/berita/1084-pertemuan-pakar-falak-mabims-2019.
[5] Mohammad Ilyas (1997), *Sistem Kalendar Islam dari Perspektif Astronomi*, p. 51.
[6] Abdul Haq Sultan (2007), "First Visibility Of The Lunar Crescent: Beyond Danjon's Limit," *The Observatory*, Vol. 127, pp. 53–59.
[7] Amir Hasanzadeh (2012), "Study of Danjon Limit in Moon Crescent Sighting," *Astrophysics and Space Science*, Vol. 339, No. 2, pp. 211–221.
[8] Nazhatulshima Ahmad etl al. (2020), "A New Crescent Moon Visibility Criteria using Circular Regression Model: A Case Study of Teluk Kemang, Malaysia," *Sains Malaysiana*, Vol. 49, No. 4, pp. 859–870.

not sighted, the current Hijri month would be completed to 30th day, and the new Hijri month is commenced the day after.[9] This is based on al-Hadith:

> Don't you start fasting until you witness the Hilal, and don't you break your fast until you witness the Hilal. In the event of cloudy, count it.[10]

Ibn Hajar Al-Asqalani argues that this hadith states that the determination of next Hijri month is by the sighing of Lunar Crescent at every 29th of current Hijri Month, and in the event that the Lunar Crescent is not sighted, either due to cloud or hindrance, the current Hijri Month is completed to 30th day.[11] Muslim worldview practice Fasting, and celebrate Eid Fitr and Eid Aidha, yearly, which denotes by the month of Ramadhan, Shawal and Zulhijjah. Ramadhan, Shawal dan Zulhijjah require new moon visibility to commenced, therefore Muslim take a considerable interest on lunar crescent visibility research. This is the reason why Schaefer concedes that lunar crescent visibility research is the most non-trivial study in Astronomy.[12]

Middle Age Lunar Crescent Visibility Criterion

Medieval Lunar crescent Visibility Criterion is criterion that found during the period of 500 A.D to 1901 AD. The study of lunar crescent visibility is heavily influenced by the work of Indian, Parsi, and Greek literature of astronomy. Al-Arjabar, Zij al-Arkand, Zij al-Sindhind, Zij ash-Shah, and Ptolemy's al-Magest is one of the few translated astronomy literatures that set the background Medieval Astronomy.[13] Following the works of their spiritual astronomy predecessor, majority of Medieval Muslim Astronomer compiled their lunar crescent visibility criterion in the form of handbook or zijes.[14] Most of them also follow the similar planetary theory, by basing the astronomical calculation using ecliptic coordinate and zodiacal longitude, while some of the lunar crescent visibility criterion follows horizontal coordinate, such as elongation to describe lunar crescent parameter. Their equation of the lunar crescent visibility limit can be summarized with equation (1).

$$\boldsymbol{\Delta\lambda + \mu\beta > f(n)} \tag{1}$$

Where $\Delta\lambda$ is the different between lunar and solar longitude, μ is variable that dependent on the location latitude, β is lunar latitude, and $f(n)$ is the series of the longitude limit. Most of the work on lunar crescent visibility in the Medieval are based on theoretical result, thus explaining the low level of their accuracy.[15]

During the Middle Ages, in the period of 5th to 15th centuries, there was a peak on the study of science, logic and history. The peak is due to the decline on scientific culture from their western contemporary, in addition to the patronage by the leader and caliph that encourage openness in scientific inquiry and exploration in scientific research. The study of lunar crescent visibility criterion also impacted by the ascension of scientific study during Middle Age among Muslim.[16] Khwarizmi believed to be the pioneer in the study of lunar crescent visibility research in 8th century, and sparks intense examination of lunar crescent visibility that lasted for few centuries until Nasirudin Al-Tusi lunar crescent visibility research in 13th century and birth to at least seven lunar crescent visibility criteria, before reaching that state of stagnation until 20th century.[17]

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[9] Nur Nafhatun Md Shariff, Muhamad Syazwan Faid & Zety Sharizat Hamidi (2016), "Islamic New Moon Software: Current Status," in *Information Science and Applications (ICISA)*, Singapore: Springer, pp. 1069–1079.
[10] Abd Allah Muhammad b. Isma'il Abu al-Bukhari (1403H), *al-Jami' al-Sahih*, [Kitab Saum]no. hadis 1909, Kaherah: al-Matba'ah al-Salafiyyah.
[11] Ibn Hajar Al-'Asqalani (1993), *Fath al-bari bi-sharh sahih al-Bukhari: wa-ma'ahu tawjih al-qari 'ila al-qawa'id wa-fawa'id al-usuliyah wa-al-hadithiyah wa-al-isnadiyah*, A. al-'Aziz ibn 'Abd A. I. Baz. (ed.), Beirut: Darul Fikir, p. 412.
[12] Bradley E. Schaefer (1997), "Crescent Wars," *Bulletin of the American Astronomical Society*, Vol. 29, p. 1206.
[13] Regis Morelon (1996), "Eastern Arabic Astronomy Between the Eighth and the Eleventh Centuries," in R. Rashed (ed.), *Encyclopedia of the History of Arabic Science*, London: Routledge, pp. 20–57.
[14] Edward Stewart Kennedy (1956), *A Survey of Islamic Astronomical Tables*, Philadephia: American Philosophical Society, pp. 123–177.
[15] Louay J. Fatoohi (1998), "First Visibility of the Lunar Crescent and Other Problems in Historical Astronomy," PhD Dissertation, Durham University, p. 712.
[16] David A. King (1987), "Some Early Tables for Determining Lunar Crescent Visibility," *Anuals of the New York Academy of Science*, Vol. 500, No. 1, pp. 185–225.
[17] Fatoohi (1998), "First Visibility of the Lunar Crescent," p. 234.

At the end of 19th century until early of 20th century saw a rekindling interest in study of Science during the Middle Ages, with David King, Kennedy, and Hogendjik taking the front seat publishing numbers of research articles, books and works, on Middle Age lunar crescent visibility research.[18] While the frequency of publication on Middle Ages Lunar crescent visibility research has dwindle in the 21st century, there is still ongoing research on Middle Ages lunar crescent visibility research such as Fatoohi and Yazdi.[19] Review on Middle Age mathematics and astronomy is widely conducted by researcher,[20] however a modern review on Middle Age lunar crescent visibility criterion is produced in limited capacity. Considering the research gap on this subject matter, this articles endeavor to provide a review of Middle Ages lunar crescent visibility criterion, that published by the researcher during the 19th until 21st century. The endeavor is prompted by the limited presence of a literature work that review the Middle Ages lunar crescent visibility criterion as a whole and provide a new light on comparative work of Middle Age lunar crescent visibility criterion.

Methodology

This articles exercise two methods in examining the Middle Ages lunar crescent visibility criterion. The two methods are literature selection and literature review. A literature search was conducted using the Google Scholar, Scopus, Mendeley and Web of Science database. For the selection of keywords, the term middle, ages, medieval, lunar, crescent, visibility, criterion was used. A snowball literature search is also employed as most of the publication on Middle Ages lunar crescent visibility criterion does not primarily used the term Middles Ages or Medieval. A literature search is supplemented by snowball techniques to cover more ground on Middle Age lunar crescent visibility criterion search. The snowball technique is conducted by assessing the "related articles" on database and referencing the bibliography of a literature on Middle Age lunar crescent visibility criterion, while adhering to the inclusion and exclusion criterion.

Publication that able to form a mathematical equation from a study on Middle Ages lunar crescent visibility table or criterion is proceeded to assessment part. While Publication that does not form any mathematical equation is excluded from assessment part. Middle Ages lunar crescent visibility criterion predominantly employ a different astrometry library from modern astrometry library. This is because Middle Ages astrometry calculation using different ephemeris, epoch, calculation formulae, celestial object point of reference, and highly dependent on latitude of the creator, in comparison to modern astrometry library. Extracting a mathematical equation from a Middle Age lunar crescent visibility criterion is an arduous task, could be prone to error without proper research and examination. Therefore, this article exclude publication that does not provide the mathematical equation of a Middle Age lunar crescent visibility criterion for the assessment part.

The literature search also specifically for English language since most of literature works is translated into English. Works that published other language is not included since it requires translation and could

[18] Kennedy (1956), *A Survey of Islamic Astronomical Tables*, p. 14; King (1987), "Some Early Tables for Determining Lunar Crescent Visibility," p. 125; David A. King (1988), "Ibn Yunus On Lunar Crescent Visibility," *Journal of History of Astronomy*, Vol. 19, No. 3, p. 4; David A. King (1993), *Astronomy in the service of Islam*, London: Routledge, p. 478; Edward Stewart Kennedy (1986), "The Lunar Visibility Theory of Ya'qub Ibn Tariq," *Journal Of Near Eastern Studies*, Vol. 27, No. 2, pp. 126–132; Edward Stewart Kennedy & Mardiros Janjanian (1966), "The Crescent Visibility Table in Al-Khwārizmīs Zīj," *Centaurus*, Vol. 11, No. 2, p. 11; Edward Stewart Kennedy & Jan P. Hogendijk (1988), "Two Tables from an Arabic Astronomical Handbook for the Mongol Viceroy of Tibet," in E. Leichty & M. de J. Ellis (eds.), *A Scientific Humanist, Studies in Memory of Abraham Sachs*, Pennsylvenia: The University Museum, pp. 233–242.

[19] Fatoohi (1998), "First Visibility of the Lunar Crescent," p. 111; S. S. H. Rizbvi (1974), "Al-Biruni Method of Visibility and its Modern Version," *International Congress on History of Science*, p 42; Hamid-Reza Giahi Yazdi (2003), "Nasir al-Dīn al-Tūsī on Lunar Crescent Visibility and an Analysis with Modern Altitude-Azimuth Criteria," *Suhayl. International Journal for the History of the Exact and Natural Sciences in Islamic Civilisation*, Vol. 3, pp. 231–243; Hamid-Reza Giahi Yazdi (2009), "Al-Khāzinī's Complex Tables for Determining Lunar Crescent Visibility," *Suhayl. International Journal for the History of the Exact and Natural Sciences in Islamic Civilisation*, Vol. 9, p. 27.

[20] Glen Van Brummelen (2014), "A Survey of Research in the Mathematical Sciences in Medieval Islam from 1996 to 2011," in Nathan Sidoli & Glen Van Brummelen (eds.), *From Alexandria, Through Baghdad*, Berlin: Springer, pp. 101–138. David A. King (2000), "Mathematical Astronomy in Islamic Civilisation," in Helaine Selin & Sun Xiaochun (eds.), *In Astronomy Across Cultures*, Dordrecht: Springer, pp. 585–613; J. Lennart Berggren (2014), "Mathematics and her sisters in medieval Islam: A selective review of work done from 1985 to 1995 [1997]," in Nathan Sidoli & Glen Van Brummelen (eds.), *From Alexandria, Through Baghdad*, Berlin: Springer, p. 412. J. Lennart Berggren (1985), "History of Mathematics in the Islamic World: The Present State of the Art," *Middle East Studies Association Bulletin*, Vol. 19, No. 1, pp 344-364; David A. King (1991), "Lunar Crescent Visibility Predictions in Medieval Islamic Ephemerides," in S. M. Seikaly, R. Baalbaki & P. Dodd (eds.), *Quest for Understanding: Arabic and Islamic Studies in Memory of Malcolm H. Kerr:* Vol. IV, Beirut: American University of Beirut, p. 2491; Frans Bruin (1977), "The First Visibility of the Lunar Crescent," *Vistas in Astronomy*, Vol. 21, pp. 331–358; Mohd Saiful Anwar Mohd Nawawi et al. (2015), "Sejarah Kriteria Kenampakan Anak Bulan di Malaysia," *Journal of Al-Tamaddun*, Vol. 10, No. 2, pp. 61–75; Mohd Saiful Anwar Mohd Nawawi et al. (2020), "Pemikiran Imam Taqī Al-Dīn Al-Subkī (683/1284-756/1355) Berkaitan Kriteria Kenampakan Anak Bulan," *Jurnal Syariah*, Vol. 28, No. 1, pp. 1–30; Nurul Kausar Nizam et al. (2014), "Penggunaan Teleskop: Kesan Terhadap Hukum Ithbat Kenampakan Anak Bulan di Malaysia," *Jurnal Fiqh*, Vol. 11, pp. 55–74.

lead to misinterpretation. The works of Middle Age lunar crescent visibility criterion must publish in indexed journal or publisher, with peer review requirement. The works on Middle Age lunar crescent visibility criterion that has not but published by any indexed journal is excluded. Since this article endeavor to provide review and assessment of a Middle Age lunar crescent visibility criterion, publication must discuss about the mathematical part of a lunar crescent visibility criterion. Publication that only discuss about the existence, history and theory of a Middle Age lunar crescent visibility criterion is excluded. The study on Middle Age lunar crescent visibility criterion also limited to lunar crescent visibility criterion that published under the patron of Muslim leadership, or by Muslim astronomer or produced for the purpose of Islamic practice, this means that works such as lunar crescent visibility criterion that produced by Jewish Middle Age astronomer such as Maimonides, is excluded.

Review of Islamic Middle Age Lunar Crescent Visibility Criterion

A total of 27 literature are collected in the first inclusion and exclusion criterion. Snowballing technique shows another 9 literature that pertains to Middle Ages lunar crescent visibility criterion, totaling of 36 literatures on Middles Ages lunar crescent visibility criterion. 15 literatures are rejected as it does not discuss the mathematical part of the lunar crescent visibility criterion, and another 8 literatures are rejected since it does not write in English. The flowchart of the inclusion & exclusion criterion is portrayed in Figure 1 while literature that pass the inclusion and exclusion criterion are as is provided in Table 1.

Table 1: Selected Literature for Middle Age Lunar Crescent Visibility Criterion Review

No	Literature	Criterion Included	Methodology
1	Fatoohi, Louay,1998, First visibility of the lunar crescent and other problems in historical astronomy	Al-Khawarizmi, Al-Qallas, al-Lathiqi, al-Sanjufini, ibn Yunus, Thabit Ibn Qurra, Ya’qub ibn Tariq, Maimonides	Literature Comparison, Error Rate Analysis
2	David A. King, 1988, Ibn Yūnus on Lunar Crescent Visibility	Ibnu Yunus	Mathematical Examination
3	Jan P. Hogendijk, 1988, Three Islamic Lunar Crescent Visibility Tables	Al-Khawarizmi, Muhamad ibn Ayyub al Tabari, Abu Jaa’far al-Khazini	Mathematical Examination
4	Hamid-Reza Giahi Yazdi, 2018, The mysterious table of lunar crescent visibility attributed to Al-Bīrūnī and Ḥabash Al-Ḥāsib’s contribution	Al-Biruni	Mathematical Examination
5	David A. King, 1987, Some Early Islamic Tables for Determining Lunar Crescent Visibility	Al-Khawarizmi, Al-Qallas, Andalusian Crescent Visibility Tables, Abu Jaa’far al-Khazini, al-Biruni, Al-Zarqallu, Al-Ladhiqi,	Mathematical Examination
6	E. S. Kennedy, Mardiros Janjanian, 1966, The Crescent Visibility Table in Al-Khwārizmīs Zīj	Al-Khwārizmīs	Mathematical Examination
7	Jan P. Hogendijk, 1988, New Light on the Lunar Crescent Visibility Table of Yaʿqūb ibn Ṭāriq	Yaʿqūb ibn Ṭāriq	Mathematical Examination
8	Ilyas, 1994, Lunar Crescent Visibility Criterion and Islamic Calendar	Yaqub Ibn Tariq, Al Biruni,	Mathematical Examination,

			Literature Comparison
9	Hamid-Reza Giahi Yazdi, 2009, Al-Khāzinī's Complex Tables for Determining Lunar Crescent Visibility	Al-Khāzinī's	Mathematical Examination
10	Hamid-Reza Giahi Yazdi, 2002, Nasir al-Dīn al-Tūsī on Lunar Crescent Visibility and an Analysis with Modern Altitude-Azimuth Criteria	Nasir al-Dīn al-Tūsī	Mathematical Examination, Modern Literature Comparison
11	Robert G. Morrison, 2019, Tables for Computing Lunar Crescent Visibility in Adderet Eliyahu	Elijah Bashyatchi	Mathematical Examination
12	Frans Bruin, 1977, The first visibility of the lunar crescent	Al-Battani	Mathematical Examination
13	E. S. Kennedy, 1968, The Lunar Visibility Theory of Yaʿqūb Ibn Ṭāriq	Yaʿqūb Ibn Ṭāriq	Mathematical Examination

Analysis Methodology of a Middle Age Lunar Crescent Visibility Criterion

Literature Analysis

A literature analysis of a lunar crescent visibility criterion is an analysis based on the qualitative assessment of a lunar crescent visibility criterion in comparison to other criterion according to the available literature. A literature analysis enables to demonstrate a glimpse assessment of a lunar crescent visibility criterion reliability and practicality. Example is the assessment of Nasirudin al-Tusi lunar crescent visibility criterion.[21] While al-Tusi used a different parameter for his lunar crescent visibility tables, one can draws the similarity, and assesses both weaknesses and strength of al-Tusi lunar crescent visibility table based on under the framework of modern lunar crescent visibility criterion, such as altitude-azimuth lunar crescent visibility criterion of Maunder and Fotheringham.[22] This provided that al-Tusi calculation reference for latitude and obliquity can be produced. A literature analysis gave a comparable performance of al-Tusi lunar crescent visibility criterion in predicting lunar crescent visibility with modern lunar crescent visibility criterion.

Error Rate Analysis

Error rate analysis is an extension of moon sighting records analysis. While moon sighting records analysis is based on limited data, error rate analysis is an analysis on lunar crescent visibility criterion based on whole collection of moon sighting database. Error rate analysis enable to demonstrate the reliability and practicality of a lunar crescent visibility criterion, as it is based on large amount of data. This means that is able to demonstrate the criterion performance at various lunar crescent geometrical parameter and geographical location. The error rate analysis in divided into two, positive error rate analysis, and negative error rate analysis. Positive error rate analysis is an analysis where lunar crescent is predicted by the criterion to be sighted but invisible in actual observation. The positive error rate is calculated through the formula of,

$$\frac{Total\ Positive\ Moon\ Sighting\ Data - Number\ of\ Positive\ Sighting\ Error}{Total\ Positive\ Moon\ Sighting\ Data}\ X\ 100 \qquad (2)$$

Negative error rate analysis is an analysis where lunar crescent is predicted by the criterion to be not sighted but visible in actual observation. The negative error rate is calculated through the formula of,

$$\frac{Total\ Negative\ Moon\ Sighting\ Data - Number\ of\ Negative\ Sighting\ Error}{Total\ Negative\ Moon\ Sighting\ Data}\ X\ 100 \qquad (3)$$

[21] Yazdi (2003), "Nasir al-Dīn al-Tūsī on Lunar Crescent Visibility," pp. 231–243.

[22] Edward Walter Maunder (1911), "On the Smallest Visible Phase of the Moon," *The Journal of the British Astronomical Association*, Vol. 21, pp. 355–362; John Knight Fotheringham (1910), "On The Smallest Visible Phase of the Moon," *Monthly Notices of the Royal Astronomical Society*, Vol. 70, No. 7, p. 527; John Knight Fotheringham (1921), "The Visibility of the Lunar Crescent," *The Observatory*, Vol. 44, pp. 308–311.

Fatoohi are responsible for introducing the Error Rate Analysis in lunar crescent visibility criterion.[23] Before Fatoohi works on lunar crescent visibility criterion, the analysis of lunar crescent visibility criterion is conducted through limited moon sighting data analysis. The weakness of the error rate analysis is that it requires large amount of data. Fatoohi amassed a total of 507 data of moon sighting for conduct the error rate analysis. In addition, the number of positive and negative moon sighting must be balanced or at least in ratio of 1 to 3 to ensure a fair analysis on both positive and negative error rate.

Mathematical Discussion

A mathematical discussion of a lunar crescent visibility criterion is an analysis based on the mathematical foundation of a lunar crescent visibility criterion. A mathematical discussion able to extract planetary theory which a lunar crescent visibility criterion is based on, in addition to its latitude, obliquity and parameter reference. An examination of mathematical discussion also led to study of an influence or origin of a lunar crescent visibility criterion. Such example for mathematical discussion is al-Khawarizmi and al-Qallas lunar crescent visibility criterion.[24] While al-Khawarizmi and al-Qallas lived 300 years apart, al-Qallas demonstrate influence from al-Khawarizmi lunar crescent visibility criterion. Despite a 300 years difference, al-Qallas unable to provide a significant improvement over al-Khawarizmi prediction for lunar crescent visibility, indicating a weakness for al-Khawarizmi design for lunar crescent visibility prediction.

Publication Chronology of Middle Lunar Crescent Visibility Criterion

The publication of lunar crescent visibility criterion can be found throughout the Middle Age. A rule or handbook for a lunar crescent visibility first produced by al-Khawarizmi in 8th century, using solar longitude and arc of separation parameter. Solar longitude and arc of separation parameter found it footing in al-Qallas, al-Tusi, and al-Lathiqi lunar crescent visibility prediction. This either due to the simplicity of arc of separation parameter, similarity of planetary theory or strong influence of al-Khawarizmi lunar crescent visibility table. An al-Khawarizmi contemporaries, Ya'qub ibn Tariq published a lunar crescent visibility rule, using angular distance parameter, a parameter that does not reflect planetary theory of his century, which primary favor longitude or latitude in describing lunar crescent visibility. Angular separation or elongation found its application in Ibn Qurra, Ibn Yunus and Elijah Bašyazi in later centuries, and become an important parameter in lunar crescent visibility criterion in modern century. A list of lunar crescent visibility criterion based on its produced year is portrayed in Table 2.

Table 2: Lunar Crescent Visibility Criterion based on Century, Author, and Parameter

Century	Author	Parameter
8th	Al-Khawarizmi	Solar Longitude, Arc of Separation
8th	Ya'qub Ibn Tariq	Angular Distance, Arc of Separation
9th	Thabit bin Qurra	Angular Distance, Solar Depression
10th	Al-Qallas	Solar Longitude, Arc of Separation
11th	Ibn Yunus	Angular Distance, Arc of Separation, Lunar Velocity
11th	Al-Biruni	Solar Longitude, Moon Latitude, Arc of Separation
12th	Al-Khazini	Solar Longitude, Arc of Separation, Lunar Velocity
13th	Nasirudin Al-Tusi	Solar Longitude, Arc of Separation
13th	Maimonides	Moon Longitude, Arc of Separation,
14th	Al-Sanjufini	Sun-Moon Longitude, Arc of Separation
15th	Elijah Bašyazi	Angular Distance, Arc of Separation
17th	Al-Lathiqi	Solar Longitude, Arc of Separation

The earliest recorded publication on Middle Age lunar crescent visibility criterion can be found in 1880, a German language work on Maimonides lunar crescent visibility rule by Israel Hildesheimer.[25] Early research on historical astronomy is primarily published in German language,[26] until Edward Kennedy

[23] Fatoohi (1998), "First Visibility of the Lunar Crescent," p. 823.

[24] Fatoohi (1998), "First Visibility of the Lunar Crescent," p. 998, King (1987), "Some Early Tables for Determining Lunar Crescent Visibility," pp. 185–225; King (1991), "Lunar Crescent Visibility Predictions in Medieval Islamic Ephemerides," p. 601.

[25] Moses Maimonides & Ezriel Hildesheimer (1881), *Die astronomischen Kapitel in Maimonidis: Abhandlung über die Neumondsheiligung*, Berlin: Driesner, p. 514.

[26] Maimonides & Hildesheimer (1881), *Die astronomischen Kapitel in Maimonidis*, p. 519; Bernhard Cohn (1918), *Der Almanach perpetuum des Abraham Zacuto: Ein Beitrag zur Geschichte der Astronomie im Mittelalter*, Strassburg: Karl J. Trübner, p. 2001; Eduard Baneth (1898), *Maimuni's Neumondsberechnung*, Berlin: Itzkowski p. 344; Eduard Mahler (1889), *Maimonides' Kiddusch hachodesch : hilkhot kidush ha-*

published his work on Thabit Ibn Qurra lunar crescent visibility table in 1960.[27] Early 20th century saw a peak on Middle Age lunar crescent visibility criterion work, with King, and Kennedy published majority of the work and instrumental in modern formation of lunar crescent visibility criterion[28]. Figure I demonstrates the frequency of published works on Middle Age lunar crescent visibility criterion that pass the selection criteria.

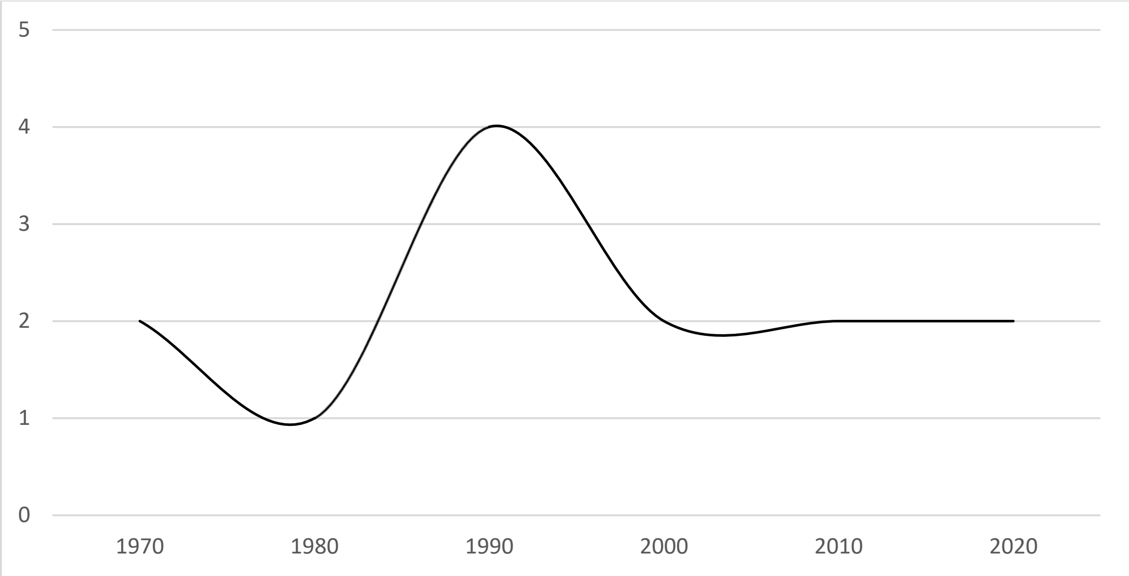

Figure 1: Graph of Publication Frequency against Year of Publication on Middle Age Lunar Crescent Visibility Criterion

Review for Middle Age Lunar Crescent Visibility Criterion

Al-Khawarizmi's Lunar crescent Visibility Criterion

Al-Khawarizmi, a famous astronomer from Baghdad born in 780, has recorded a lunar crescent visibility criterion in a part of a handbook. His table of lunar crescent visibility criterion is called al-ru'ya lil Khawarizmi, meaning the visibility according to al- Khawarizmi.[29] Al-Khawarizmi uses two main parameters in this criterion, zodiacal sign, referring to the value of longitude, and the difference of longitude between sun and moon, $\Delta\lambda$. The lunar crescent will be visibility when its difference in sun-moon longitude exceeds the value of $\Delta\lambda$. Table of al-Khawarizmi lunar crescent visibility Criterion portray in Table 3.

The value of $\Delta\lambda$ can be interpreted as an arc of separation or lag time between sunset and moonset. This indicates Indian criterion influence on Khawarizmi lunar crescent visibility criterion. Fatoohi, when tested al-Khawarizmi lunar crescent visibility criterion on 33-degree latitude, has found out that al-Khawarizmi criterion has error rate of 31.6% in predicting invisibility of lunar crescent and error rate of 9.6% percent in predicting the visibility of lunar crescent.[30]

The error rate rises significantly when tested with other degrees of latitude, demonstrating the unreliability of the criterion in predicting the first visibility of lunar crescent. Khwarizmi Criterion, despite its unreliability, indicating a relationship between the position of the earth in respect to sun with the lag time in determining visibility of lunar crescent can be constructed.

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hodesh, Vienna: Verag von Adolf Fanto, p. 752; Carlo Alfonso Nallino (1899), *Al-Battānī sive Albatenii Opus Astronomicum*, Milan: Ulrico Hoepli, p. 843.

[27] Edward Stewart Kennedy (1960), "The Crescent Visibility Theory of Thabit bin Qurra," *Proceedings of the Mathematical and Physical Society of the United Arab Republic*, Vol. 24, pp. 71–74.

[28] Kennedy (1956), *A Survey of Islamic Astronomical Tables*, pp. 123–177; King (2000), "Mathematical Astronomy in Islamic Civilisation," pp. 585–613.

[29] King (1987), "Some Early Tables for Determining Lunar Crescent Visibility," pp. 185–225.

[30] Fatoohi (1998), "First Visibility of the Lunar Crescent," p. 532.

Table 3: Al-Khawarizmi Lunar Crescent Visibility Table

Solar Longitude	Arc of Separation	Solar Longitude	Arc of Separation
30	10 ˚12’	*210*	18 ˚ 36’
60	9 ˚ 58’	*240*	16 ˚ 07’
90	10 ˚ 01’	*270*	12 ˚ 58’
120	11˚ 23’	*300*	10 ˚ 40’
150	14 ˚ 29’	*330*	9 ˚ 56’
180	17 ˚ 44’	*360*	10 ˚ 04’

Ya’qub Ibn Tariq Lunar crescent Visibility Criterion

Yaʿqūb ibn Ṭāriq was an 8th-century Persian astronomer and mathematician who lived in Baghdad. Ibn Tariq provide a unique method in lunar crescent visibility criterion. While majority of the Medieval lunar crescent visibility criterion follows the Indian arc of separation condition in lunar crescent visibility table, there are instances where Medieval lunar crescent visibility criterion has conditioned Indian rule of visibility to increase its accuracy. Ya’qub Ibn Tariq, a prominent astronomy in the 8th century, has produced a lunar crescent visibility condition that twinned with the parameter of elongation.[31] Ya’qub Ibn Tariq condition is that moon will be visible if it satisfy the following condition,

Lag time ≥ 12° and Elongation > 11.25°.
Lag time ≥ 12° and Elongation > 15°.

Fatoohi, in examining Ibn Tariq condition has found an astonishing 4.6% error rate in predicting the invisibility and visibility of the lunar crescent for latitude of 30-35°, while in all latitude, it yields error rate of 28.6% for negative observation and 6.9% for positive observation.[32] Ibn Tariq lunar crescent visibility criterion is the first to criterion that introduce the inclusion of elongation in its parameter. Elongation parameter is the used by Ibn Qurra century later and popularized by Danjon in 1936, 9 centuries later. Ibn Tariq inclusion of elongation has revolutionized the modern lunar crescent visibility criterion as we have known today.[33]

Ibn Yunus Lunar Crescent Visibility Criterion

Ibnu Yunus is celebrated astronomer from Cairo around 1000 AB. He is among the first Muslim scientist that suggest a multi-parameter criterion for lunar crescent visibility prediction. Ibnu Yunus incorporate three parameters in determining lunar crescent visibility, first would be parameter of sun-moon angular distance, or elongation, which he interpreted as lunar crescent width.[34] Ibn Yunus second parameter is lag time interpreted as windows of successful moon sighting and third is earth-moon distance which interpreted as brightness of the moon. Ibnu Yunus table, in comparison to al-Khawarizmi table, is based condition. If the moon passes the certain logic condition, it predicted to be visible or invisible. Ibnu Yunus visibility table is expressed as Table 4.

Table 4: Ibnu Yunus Lunar Crescent Visibility Table

Condition			**Visibility**
Angular Distance / Elongation	Arc of Separation	Lunar Velocity	
12	15	slow	No
13	15	slow	No
14°	15°	slow	Yes
15	10	slow	Yes
11	>15	fast	No
13	10	fast	No
12°	15°	fast	Yes

[31] Kennedy (1986), “The Lunar Visibility Theory of Ya’qub Ibn Tariq,” pp.126–132.
[32] Fatoohi (1998), “First Visibility of the Lunar Crescent,” p. 764.
[33] Jan P. Hogendijk (1988b), “New Light on the Lunar Crescent Visibility Table of Ya’qub Ibn Tariq,” *Journal of Near Eastern Studies*, Vol. 47, No. 2, pp. 95–104.
[34] King (1988), “Ibn Yunus On Lunar Crescent Visibility,” p. 73

13	<15	fast	Yes
14	10	fast	Yes
11	>15	max.	Yes
11	<15	max.	No

Ibn Yunus conditional style of visibility table in addition to limited value of elongation and lag time render its reliability examination to be difficult.[35] The study can be conducted in a limited manner encompassing small number of lunar crescent visibility report.

Thabit Ibn Qurra Lunar crescent Visibility Criterion

In addition to Ibnu Yunus composite criterion, there is another astronomer that produced composite parameter to determine the lunar crescent visibility, which is Thabit bin Qurra. Thabit Bin Qurra is Baghdad astronomer, active during the ninth century during the period of Abbasid Caliphate, is one of the most influential individuals in understanding and reforming the original geocentric model of the universe.[36] Aligning with his geocentric model reform, Thabit Ibn Qurra does not follows that footstep of using ecliptic longitude and latitude for lunar crescent visibility determination, instead he uses elongation, solar depression, and difference in azimuth.[37] The Ibn Qurra lunar crescent visibility criterion is as follows,

Elongation < 10.8°: Lunar crescent is not sighted
Elongation > 25°: Lunar crescent is sighted
10.8° ≤ Elongation ≤ 25° and solar depression ≥ 11.1°: Lunar crescent is sighted
10.8° ≤ Elongation ≤ 25 and solar depression < 11.1°, the visibility is subjected to compliated calculation

Fatoohi found out that Ibn Qurra criterion has incorrectly predict at least 20% of negative observation, indicating it is a weak criterion. Ibn Qurra criterion despite being low level of reliability, has shown a progressive evolution from its predecessor by using a horizontal coordinate.[38] Horizontal coordinate enables a more direct measurement of lunar crescent parameter, as parameter using a topocentric horizon as a reference.[39]

Al-Qallas's Lunar crescent Visibility Criterion

Maslama Ibn Ahmad al-Majriti nicknamed al-Qallas, a Cordova astronomer in the 10th century produced a lunar crescent visibility, which Ibn Ishaq al-Tunisi attributed the table to popular name, al-Qallas. The lunar crescent visibility table is revision of Khwarizmi table of lunar crescent visibility, with the division of the visibility function on every zodiacal sign into three, meaning al-Qallas assigned a visibility function at every 10-degree solar longitude.[40] The division could suggest more particular research is done to defining the limiting visibility of the lunar crescent. Kennedy argues that the Table is based on latitude of 40.87 degree to 41.35 degree with obliquity of 23.55 degree to 24.00 degree.[41] The table of lunar crescent visibility as be expressed as Table 5.

Table 5: Al-Qallas Lunar Crescent Visibility Tables[42]

Solar Longitude	Arc of Separation	Solar Longitude
10	9 ˚ 26'	190
20	9 ˚ 25'	200
30	9 ˚ 21'	210
40	9 ˚ 19'	220
50	9 ˚ 18'	230
60	9 ˚ 21'	240

[35] Fatoohi (1998), "First Visibility of the Lunar Crescent," p. 764.
[36] Kennedy (1960), "The Crescent Visibility Theory of Thabit bin Qurra," pp. 71–74.
[37] Morelon (1996), "Eastern Arabic Astronomy," pp. 20–57; Regis Morelon (1996), "General Survey of Arabic Astronomy" in R. Rashed (ed.), *Encyclopedia of the History of Arabic Science*, London: Routledge, pp. 1–19.
[38] Fatoohi (1998), "First Visibility of the Lunar Crescent," p. 819.
[39] Kennedy (1960), "The Crescent Visibility Theory of Thabit bin Qurra," pp.71–74.
[40] Kennedy (1956), *A Survey of Islamic Astronomical Tables*, pp. 123–177.
[41] Fatoohi (1998), "First Visibility of the Lunar Crescent," p. 543.
[42] Fatoohi (1998), "First Visibility of the Lunar Crescent," p. 545.

70	9 ˚ 33’	250
80	9 ˚ 57’	260
90	10 ˚ 37’	270
100	11 ˚ 29’	280
110	12 ˚ 48’	290
120	14 ˚ 15’	300
130	15 ˚ 58’	310
140	17 ˚ 31’	320
150	19 ˚ 11’	330
160	20 ˚ 20’	340
170	21 ˚ 04’	350
180	21 ˚ 17’	360

Similarly, with Khawarizmi, the lunar crescent is predicted to be visible when it exceeds the value of the visibility function $\Delta\lambda$. This table suggest at the limiting lagtime of lunar crescent visibility ranging from 37.04 minutes to 85.68 minutes according to solar longitude. Fatoohi, in his accuracy test of the Al-Qallas visibility table has found out that it has error rate of 54.5% in predicting the invisibility of lunar crescent and error rate of 4.2% in predicting the visibility of lunar crescent.[43] Al-Qallas revision of al-Khawarizmi is found out the be less accurate than the original al-Khawarizmi lunar crescent visibility table.

Al-Khāzinī’s Lunar crescent Visibility Criterion

Abu al-Fath Abd Al-Rahman al-Khazini is a celebrated astronomer who live in the Merv, currently known as Mary in Turkmenistan during the period of 1081-1131. Al-Khazini was under the patron of Sultan Sanjar bin Malikshah, thus al-Khazini has dedicated an astronomy handbook entitled al-Zij al-mu’tabar al-sanjari encompassing the result of thirty-five years of observation. It is highlighted by Kennedy that the work of Khazini on lunar crescent visibility table is the most comprehensive study on lunar crescent visibility during him time.[44]

Khazini works on lunar crescent visibility criterion is heavily influenced by Thabit Ibn Qurra, which explain why one of Khazini lunar crescent visibility table follow the same methodology of Thabit Ibn Qurra. Khazini uses the parameter of Elongation, Solar Depression, Lag time, sun-moon longitude difference, moon angular speed and lunar altitude. Khazini has two known lunar crescent visibility criteria, the first one is similar with Thabit Ibn Qurra lunar crescent visibility, using the conditional criteria framework.[45] The first Khazini lunar crescent visibility criterion is based on the lunar position in respect to Earth, with its elongation, lag time, solar depression, and sun-moon longitude. The al-Khazini for the lunar crescent visibility table is demonstrated in Table 6. Khazini then implies that lag time should be the deciding parameter while the other parameters are supplement to the criterion. The second Khazini lunar crescent visibility criterion is in the form of table. The table is categorized into three type of lunar crescent observation, general or easily visible, moderate, or moderately visible, and acute or rarely visible.

Khazini table is separated into three type observation to note the rate of successful sighting in a lunar crescent observation. Khazini second lunar crescent visibility criterion found to be similar with Khawarizmi lunar crescent visibility table, with introduction of arc of light and rate of successful in Khazini lunar crescent visibility table. Khazini table of lunar crescent visibility suggest that at a certain angular speed of the moon V_m, lunar crescent will be visible at arc of light more than the value of e_1 and will be visible before sunset if the arc of light is more than the value of e_2. A larger value of V_m, indicate a longer earth-moon distance, thus making the threshold value of e_1 and e_2 to be bigger.

Nasirudin Al-Tusi Lunar crescent Visibility Criterion

[43] Fatoohi (1998), “First Visibility of the Lunar Crescent,” p. 548.
[44] Yazdi (2009), “Al-Khāzinī’s Complex Tables for Determining Lunar Crescent Visibility,” p. 32.
[45] King (1987), “Some Early Tables for Determining Lunar Crescent Visibility,” pp. 185–225.

Muhammad ibn Muhammad ibn al-Hasan al-Tūsī was born in Tus, Iran in 1201. His is famously known as Nasirudin Al-Tusi, was an expert in various subject including astronomy, biology, and chemistry. In 1242, he compiles a collection of astronomy table in handbook called Ilkhani Zij which was dedicated for his patron, Ilkhan Hulagu. The handbook is a result of research and collaboration with astronomer at the Maragha observatory.[46]

Zij Ilkhani contain a lunar crescent visibility table that uses two parameter, sun-moon longitude and arc of separation or lag time. Similar with Al-Khazini lunar crescent visibility table, Al-Tusi designed his table in the form of categorization. However. Al-Tusi provides a different presentation by introducing the condition of the crescent categorization. The categorization is express in term of; thin, moderate, bright, visible, and quite visible. At-Tusi lunar crescent visibility table is expressed in Table 7. His lunar crescent visibility condition suggests that at sun-moon longitude less than 10 degree, the lunar crescent will be visible at lag time more than 48 minutes. For lunar crescent that has sun-moon longitude more than 10 degree, the lunar crescent is confident to be visible at lag time more than 72 minute, while the visibility of the lunar crescent will be difficult at lag time less than 52 minute. Al-Tusi work on lunar crescent visibility is among the pioneer that categorized the lunar crescent visibility based on the thickness and visibility of the new moon, a categorized which then follows by Bruin in 1976, 500 years later.[47]

Table 6: al-Khazini Lunar Crescent Visibility Table[48]

Solar Longitude (˚)	**Lunar Velocity**	**Arc of Separation**		
		Easily Visible	Maybe Visible	Rarely Visible
0	12 ˚ 06’	11 ˚ 50’	10 ˚ 45’	9 ˚ 40’
6	12 ˚ 07’	11 ˚ 49’	10 ˚ 45’	9 ˚ 40’
12	12 ˚ 09’	11 ˚ 48’	10 ˚ 44’	9 ˚ 39’
18	12 ˚ 12’	11 ˚ 47’	10 ˚ 42’	9 ˚ 38’
24	12 ˚ 15’	11 ˚ 45’	10 ˚ 40’	9 ˚ 37’
30	12 ˚ 18’	11 ˚ 42’	10 ˚ 38’	9 ˚ 34’
36	12 ˚ 22’	11 ˚ 40’	10 ˚ 36’	9 ˚ 32’
42	12 ˚ 26’	11 ˚ 37’	10 ˚ 33’	9 ˚ 30’
48	12 ˚ 30’	11 ˚ 34’	10 ˚ 29’	9 ˚ 27’
54	12 ˚ 34’	11 ˚ 30’	10 ˚ 25’	9 ˚ 23’
60	12 ˚ 38’	11 ˚ 25’	10 ˚ 21’	9 ˚ 19’
66	12 ˚ 42’	11 ˚ 20’	10 ˚ 16’	9 ˚ 15’
72	12 ˚ 47’	11 ˚ 15’	10 ˚ 11’	9 ˚ 10’
78	12 ˚ 54’	11 ˚ 10’	10 ˚ 05’	9 ˚ 06’
84	13 ˚ 02’	11 ˚ 04’	10 ˚ 00’	9 ˚ 01’
90	13 ˚ 10’	10 ˚ 58’	9 ˚ 56’	8 ˚ 57’
96	13 ˚ 18’	10 ˚ 53’	9 ˚ 52’	8 ˚ 52’
102	13 ˚ 24’	10 ˚ 47’	9 ˚ 47’	8 ˚ 48’
108	13 ˚ 30’	10 ˚ 41’	9 ˚ 42’	8 ˚ 43’
114	13 ˚ 37’	10 ˚ 35’	9 ˚ 37’	8 ˚ 39’
120	13 ˚ 43’	10 ˚ 29’	9 ˚ 32’	8 ˚ 34’
126	13 ˚ 50’	10 ˚ 24’	9 ˚ 27’	8 ˚ 30’
132	13 ˚ 56’	10 ˚ 19’	9 ˚ 24’	8 ˚ 26’
138	14 ˚ 01’	10 ˚ 15’	9 ˚ 20’	8 ˚ 22’
144	14 ˚ 05’	10 ˚ 12’	9 ˚ 16’	8 ˚ 19’
150	14 ˚ 09’	10 ˚ 09’	9 ˚ 12’	8 ˚ 16’
156	14 ˚ 13’	10 ˚ 08’	9 ˚ 10’	8 ˚ 14’
162	14 ˚ 17’	10 ˚ 05’	9 ˚ 08’	8 ˚ 13’
168	14 ˚ 21’	10 ˚ 04’	9 ˚ 07’	8 ˚ 12’
174	14 ˚ 25’	10 ˚ 02’	9 ˚ 06’	8 ˚ 11’
180	14 ˚ 27’	10 ˚ 00’	9 ˚ 05’	8 ˚ 10’

[46] Yazdi (2003), “Nasir al-Dīn al-Tūsī on Lunar Crescent Visibility,” p. 235.
[47] Yazdi (2003), “Nasir al-Dīn al-Tūsī on Lunar Crescent Visibility,” p. 240.
[48] Yazdi (2009), “Al-Khāzinī’s Complex Tables for Determining Lunar Crescent Visibility,” p. 35.

Table 7: Al-Tusi Lunar Crescent Visibility Table

Sun-Moon Longitude	**Arc of Separation**	**Condition of crescent**
10°	10°	Thin
	12°	Moderate
	14°	Bright
	16°	Visible
	18°	Easily Visible

Muhammad Ibn Ayyub al-Tabari Lunar Crescent Visibility Criterion

Abu Ja’far Muhammad Ibn Ayyub al-Tabari was a Persian Astronomer. There is not much known about this life, outside from hit astronomical tables and works, which primarily written in Persian language. Ibn Ayyub al-Tabari work on astronomical tables, is noted to be one of the oldest works on astronomy written in Persian language.[49] Ibn Ayyub al-Tabari publish an astronomical handbook called Mufrad Zij. His tables do not accompany by explanatory text on how the table is calculated, and what is its mathematical origin. Therefore, modern historian unable to thoroughly examine the mathematical structure of Mufrad Zij.[50]

Mufrad Zij portray a relationship between solar longitude and arc of separation. The values are calculated at the obliquity of 23˚51’, declination of 9˚30’, with location reference of 35˚50’ of latitude. Therefore, it can be inferred that Mufrad Zij is produced with Rhages, or currently known as Ray, Iran, or Tehran in mind. Mufrad Zij lunar crescent visibility tables have interval of 10 degree of solar longitude, with arc of separation maximum at 19˚05’ or 76.33 minutes at 160 degree of solar longitude, and minimum arc of separation at 38.4 minutes at 35 degrees of solar longitude. Table for Mufrad Zij is portray in Table 8. Al-Tabari lunar crescent visibility table demonstrates influences from al-Khwarizmi and Indian planetary theories. Al-Tabari however are more extensive by laying out the arc of separation for each 10 degrees of solar depression.

Al-Lathiqi Lunar Crescent Visibility Criterion

Muhamad al-Lathiqi is a Muslim astronomy born around 17th century. Al-Lathiqi is a Syrian astronomer and could be inferred that Lattakia to be his hometown. In 1698, al-Lathiqi published a table of lunar crescent visibility criterion, that use solar longitude and arc of separation as a parameter, a similar design with al-Khawarizmi lunar crescent visibility table.

Al-Lathiqi computation method is to compute difference in longitude between two luminaries at two thirds of an hour after sunset. Then if difference in longitude is greater than or equal to the visibility function limit, the lunar crescent will be predicted to be visible. However, if difference in longitude is less than visibility function limit, lunar crescent cannot be seen. Al-Lathiqi’s criterion which is shown in Table 9 demonstrate that the crescent cannot be seen if it is less than about 18.5 hours old and may remain invisible for up to 33 hours of moon age.

Table 8: Ibn Ayyub al-Tabari’s Mufrad Zij Lunar Crescent Visibility Table[51]

SOLAR LONGITUDE	**ARC OF SEPARATION**	**SOLAR LONGITUDE**	**ARC OF SEPARATION**	**SOLAR LONGITUDE**	**ARC OF SEPARATION**	**SOLAR LONGITUDE**	**ARC OF SEPARATION**
10	9 ˚ 46’	*100*	14 ˚ 40’	*190*	18 ˚ 03’	*280*	10 ˚ 58’
20	9 ˚ 53’	*110*	15 ˚ 55’	*200*	17 ˚ 36’	*290*	10 ˚ 39’
30	10 ˚ 02’	*120*	16 ˚ 57’	*210*	16 ˚ 28’	*300*	10 ˚ 12’
40	10 ˚ 16’	*130*	17 ˚ 54’	*220*	15 ˚ 35’	*310*	9 ˚ 58’
50	10 ˚ 34’	*140*	18 ˚ 31’	*230*	14 ˚ 35’	*320*	9 ˚ 55’
60	11 ˚ 06’	*150*	18 ˚ 59’	*240*	13 ˚ 42’	*330*	9 ˚ 50’
70	11 ˚ 44’	*160*	19 ˚ 05’	*250*	12 ˚ 51’	*340*	9 ˚ 44’

[49] Kennedy (1956), *A Survey of Islamic Astronomical Tables*, pp. 123–177.
[50] King (1987), “Some Early Tables for Determining Lunar Crescent Visibility,” pp. 185–225.
[51] Jan P. Hogendijk (1988), “Three Islamic Lunar Visibility Calendar,” *Journal of History of Astronomy* [preprint], p. 34.

80	12 ˚ 34’	*170*	18 ˚ 54’	*260*	12 ˚ 09’	*350*	9 ˚ 36’
90	13 ˚ 39’	*180*	18 ˚ 38’	*270*	11 ˚ 29’	*360*	9 ˚ 46’

Table 9: Lathiqi Lunar Crescent Visibility Table[52]

Solar Longitude (˚)	**Arc of Separation**	**Solar Longitude (˚)**	**Arc of Separation**
30	10 ˚ 08’	*210*	16 ˚ 30’
60	9 ˚ 14’	*240*	15 ˚ 27’
90	9 ˚ 09’	*270*	13 ˚ 30’
120	10 ˚ 31’	*300*	10 ˚ 22’
150	12 ˚ 26’	*330*	9 ˚ 11’
180	15 ˚ 03’	*360*	10 ˚ 25’

Since it uses the same al-Khawarizmi design, this criterion has percentages of error comparable to al-Khawarizmi unreliable criterion. al-Lathiqi criterion wrongly predict 37 negative observations to have been visible and 20 positive moonsighting to have been invisible. Al-Lathiqi’s solution is improve slightly than al-Khawarizmi lunar crescent visibility table in its positive observation’s prediction yet its error with the negative observations is greater.[53]

Sanjufini Lunar Crescent Visibility Criterion

In 1366, Abu Muhammad al-Sanjufini computed a handbook as a gift to his patron, Prince Randa, the Mongol viceroy of Tibet and a direct descendant in the seventh generation from Genghis Khan. One of the forty-two tables of the handbook is specified for predicting lunar crescent visibility. The table is computed for latitude 38; 10° which indicating, according to Kennedy and Hogendijk, that the table was computed for the second Mongol capital Yung-ch’ang fu.[54] Al-Sanjufini’s table gives the visibility function for each degree of lunar latitude for every 10° of solar longitude. Table 10 contains al-Sanjufini lunar crescent visibility table.

His table suggest that that the crescent cannot be seen before it is 14.5 hours old and that it may remain invisible even when it is some two days and a half old. Al-Sanjufini lunar crescent visibility table does not yield good results in predicting lunar crescent visibility criterion, with. 8.1% and 35.7% of the positive and negative observations, respectively, in contradiction with the criterion. It is interesting to note that the introduction of the lunar latitude as an additional parameter in the criterion has not improved this solution significantly in comparison with the other models which neglect the lunar latitude.[55]

Table 10: Sanjufini Lunar Crescent Visibility Criterion[56]

Solar Longitude (˚)	**Arc of Separation**			**Solar Longitude (˚)**	**Arc Of Separation**		
	ß=5°	ß=0	ß=-5°		ß=5°	ß=0	ß=-5°
0	8 ˚ 37’	9 ˚ 39’	10:43°	180	12 ˚ 50’	19 ˚ 39’	29 ˚ 18’
10	8 ˚ 30’	9 ˚ 37’	10;59	190	12 ˚ 25’	19 ˚ 0’	28 ˚ 11’
20	8 ˚ 19’	9 ˚ 38’	11:16	200	11 ˚ 55’	18 ˚ 08’	26 ˚ 45’
30	8 ˚ 07’	9 ˚ 41’	11;35	210	11 ˚ 12’	17 ˚ 03’	25 ˚ 05’
40	7 ˚ 54’	9 ˚ 45’	12:09	220	10 ˚ 37’	15 ˚ 57’	23 ˚ 13’
50	7 ˚ 41’	9 ˚ 44’	12;52	230	9 ˚ 39’	14 ˚ 45’	21 ˚ 20’

[52] Fatoohi (1998), “First Visibility of the Lunar Crescent,” p. 387.
[53] Fatoohi (1998), “First Visibility of the Lunar Crescent,” p. 390.
[54] Kennedy & Hogendijk (1988), “Two Tables from an Arabic Astronomical Handbook,” pp. 233–42.
[55] Fatoohi (1998), “First Visibility of the Lunar Crescent,” p. 457.
[56] Fatoohi (1998), “First Visibility of the Lunar Crescent,” p. 360.

60	7 ˚ 28’	10 ˚ 15’	13;59	240	9 ˚ 28’	13 ˚ 33’	19 ˚ 22’
70	7 ˚ 25’	10 ˚ 43’	15:18	250	9 ˚ 18’	12 ˚ 34’	17 ˚ 37’
80	7 ˚ 31’	11 ˚ 25’	16:43	260	8 ˚ 15’	11 ˚ 41’	16 ˚ 03’
90	7 ˚ 56’	12 ˚ 22’	18:46	270	8 ˚ 00’	10 ˚ 57’	14 ˚ 34’
100	8 ˚ 09’	13 ˚ 28’	20;29	280	8 ˚ 01’	10 ˚ 16’	13 ˚ 42’
110	9 ˚ 20’	14 ˚ 48’	22;53	290	7 ˚ 51’	10 ˚ 0’	12 ˚ 38’
120	10 ˚ 9’	16 ˚ 8’	25:4	300	7 ˚ 58’	9 ˚ 46’	12 ˚ 0’
130	10 ˚ 53’	17 ˚ 28’	27;20	310	8 ˚ 18’	9 ˚ 31’	11 ˚ 38’
140	11 ˚ 41’	18 ˚ 26’	28;37	320	8 ˚ 15’	9 ˚ 37’	11 ˚ 18’
150	12 ˚ 36’	19 ˚ 21’	29;52	330	8 ˚ 22’	9 ˚ 36’	11 ˚ 1’
160	12 ˚ 53’	19 ˚ 53’	30;31	340	8 ˚ 30’	9 ˚ 35’	10 ˚ 54’
170	12 ˚ 57’	19 ˚ 53’	30;23	350	8 ˚ 39’	9 ˚ 33’	10 ˚ 50’

Conclusion

A review on lunar crescent visibility criterion is conducted. The review found that the study of lunar crescent visibility criterion has been dwindled since in peak in early 20th century. This perhaps can be explained as most of the works on Middle Age lunar crescent visibility criterion has been published by David King, and Edward Kennedy. In addition, some of unexplored Middle Age lunar crescent visibility tables are not accompanied with explanation on the text and not claimed by any author, making the process of discovering and studying these mystery Middle Age lunar crescent visibility tables to be stagnant and in decline.

However, the review suggests that an assessment of Middle Age lunar crescent visibility criterion under framework on new data of moonsighting, in addition to a comparative work between Middle Age lunar crescent visibility criterion against modern lunar crescent visibility criterion could reinvigorate the study on Middle Age lunar crescent visibility criterion. This is because while recomputing Middle Age lunar crescent visibility to fit modern moonsighting data and lunar crescent visibility criterion would be an arduous task, process is much easier due to the existence of extensive python libraries, and the availability of moonsighting data in 21st century, in comparison during the period of David King and Edwards Kennedy.

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